CHAPTER

3 Functions

❖ **Ordered pair** Pair formed by two elemetns that are separated by a comma and written as (x, y).

❖ **Cartesian product** $A \times B$ of two sets A and B is given by

$A \times B = \{(a, b) : a \in A , b \in B\}$

In particular $R \times R = \{(x, y) : x, y \in R\}$

and $R \times R \times R = \{(x, y, z): x, y, z \in R\}$

❖ If $(a, b) = (x, y)$, then $a = x$ and $b = y$.

❖ If $n(A) = p$ and $n(B) = q$, then $n(A \times B) =$ pq.

❖ $A \times \phi = \phi$

❖ In general, $A \times B \neq B \times A$.

❖ **Relation** A relation R from a set A to a set B is a subset of the cartesian product $A \times B$ obtained by describing a relationship between the first element x and the second element y of the ordered pairs in $A \times B$.

❖ **The image** of an element x under a relation R is given by y, where $(x,y) \in R$,

❖ The **domain** of R is the set of all first elements of the ordered pairs in a relation R.

❖ The **range** of the relation R is the set of all second elements of the ordered pairs in a relation R.

❖ **Function** A function f from a set A to a set B is a specific type of relation in which every element x of set A has one and only one image y in set B.

We write $f: A \rightarrow B$, where $f(x) = y$.

❖ A is the domain and B is the codomain of f.

❖ The range of the function is the set of images.

❖ Algebra of functions For functions $f: X \rightarrow R$ and $g: X \rightarrow R$, we have

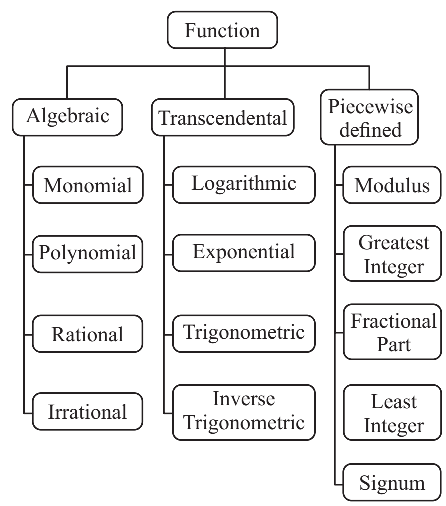
$(f + g)(x) = f(x) + g(x), x \in X$

$(f - g)(x) = f(x) - g(x), x \in X$

$(f.g)(x) = f(x) \cdot g(x), x \in X)$

$(kf)(x) = k(f(x)), x \in X$, where k is a real number.

$\left(\dfrac{f}{g}\right)(x) = \dfrac{f(x)}{g(x)}, x \in X, g(x) \neq 0$



S.No.		Transformation	How to transform
1.	(a)	$y = f(x) \rightarrow y = f(x + a)$	Shift the graph of $y = f(x)$ through 'a' units towards left.
	(b)	$y = f(x) \rightarrow y = f(x - a)$	Shift the graph of $y = f(x)$ through 'a' units towards right.
2.	(a)	$y = f(x) \rightarrow y + a = f(x)$	Shift graph of $y = f(x)$ by 'a' units downward.
	(b)	$y = f(x) \rightarrow y - a = f(x)$	Shift graph of $y = f(x)$ by 'a' units upward.
3.		$y = f(x) \rightarrow y = f(-x)$	Take the mirror image of $y = f(x)$ in the y-axis.
4.		$y = f(x) \rightarrow y = -f(x)$	Take the mirror image of $y = f(x)$ in the x-axis.
5.		$y = f(x) \rightarrow y = f(\lvert x \rvert)$	Remove the left portion of the graph after that take the mirror image of the right portion of the curve in the Y-axis. Also include the right portion of the graph of $y = f(x)$.
6.		$y = f(x) \rightarrow y = \lvert f(x) \rvert$	Take the mirror image of the lower portion of the curve (the curve below x-axis) in x-axis and reject the lower part (or flip lower part into upper).
7.		$y = f(x) \rightarrow \lvert y \rvert = f(x)$	Remove the lower portion of the curve then take the mirror image of upper portion of the curve in the x-axis. Also include the upper portion of the graph of $y = f(x)$.
8.		$y = f(x) \rightarrow y = af(x)$	Stretch ($a > 1$) or squaeeze ($a < 1$) the graph of the given function vertically.
9.		$y = f(x) \rightarrow y = f(ax)$	Stretch ($a > 1$) or squaeeze ($a < 1$) the graph of the given function horizontally.